Exercise 14

Evaluate the limit and justify each step by indicating the appropriate properties of limits.

$$\lim_{x\to\infty}\sqrt{\frac{9x^3+8x-4}{3-5x+x^3}}$$

Solution

Use the root law.

$$\lim_{x\to\infty}\sqrt{\frac{9x^3+8x-4}{3-5x+x^3}}=\sqrt{\lim_{x\to\infty}\frac{9x^3+8x-4}{3-5x+x^3}}$$

Multiply the numerator and denominator by the reciprocal of the highest power of x in the denominator.

$$\lim_{x\to\infty}\sqrt{\frac{9x^3+8x-4}{3-5x+x^3}}=\sqrt{\lim_{x\to\infty}\frac{9x^3+8x-4}{3-5x+x^3}\cdot\frac{\frac{1}{x^3}}{\frac{1}{x^3}}}$$

Multiply the fractions together.

$$\lim_{x\to\infty}\sqrt{\frac{9x^3+8x-4}{3-5x+x^3}}=\sqrt{\lim_{x\to\infty}\frac{(9x^3+8x-4)\frac{1}{x^3}}{(3-5x+x^3)\frac{1}{x^3}}}$$

Use the distributive property.

$$\lim_{x\to\infty}\sqrt{\frac{9x^3+8x-4}{3-5x+x^3}}=\sqrt{\lim_{x\to\infty}\frac{9+\frac{8}{x^2}-\frac{4}{x^3}}{\frac{3}{x^3}-\frac{5}{x^2}+1}}$$

The limit of a quotient is the quotient of the limits.

$$\lim_{x\to\infty}\sqrt{\frac{9x^3+8x-4}{3-5x+x^3}}=\sqrt{\frac{\lim_{x\to\infty}\left(9+\frac{8}{x^2}-\frac{4}{x^3}\right)}{\lim_{x\to\infty}\left(\frac{3}{x^3}-\frac{5}{x^2}+1\right)}}$$

The limit of a sum (difference) is the sum (difference) of the limits.

$$\lim_{x\to\infty}\sqrt{\frac{9x^3+8x-4}{3-5x+x^3}}=\sqrt{\frac{\lim_{x\to\infty}9+\lim_{x\to\infty}\frac{8}{x^2}-\lim_{x\to\infty}\frac{4}{x^3}}{\lim_{x\to\infty}\frac{3}{x^3}-\lim_{x\to\infty}\frac{5}{x^2}+\lim_{x\to\infty}1}}$$

Evaluate all the limits.

$$\lim_{x\to\infty}\sqrt{\frac{9x^3+8x-4}{3-5x+x^3}}=\sqrt{\frac{9+0-0}{0-0+1}}$$

Simplify the result.

$$\lim_{x\to\infty}\frac{2x^2-7}{5x^2+x-3}=\sqrt{9}$$

$$\lim_{x\to\infty}\frac{2x^2-7}{5x^2+x-3}=3$$